BMB/Bi/Ch 173 – Winter 2017

Homework Set 1.1 – Assigned 1-10-17, Due 1-17-17 by 10:30am

TA: Wen Zhou (201 Kerckoff, office hour: Fri 1/13 5-6pm, Mon 1/16 by appointment)

1. (25 points) Various Waves/Particles and Applications

In the table below,

I. In the table, place the following types of radiation next to their corresponding wavelength: microwave, x-ray, visible, radio, UV, gamma ray.

II. Using the wavelengths provided, calculate the energy of each type of radiation in joules. Hint: **E = hc/ λ**

III. Fill in the application(s) column with the following (some may be used twice): diagnostic PET scan, heating up food quickly, fluorescent light microscopy, crystallography, communication on walkie talkies.

Wavelength	Radiation	Energy (J)	Application(s)
10 km	Radio	$1.99*10^{-29}$	Walkie talkies
50 cm	Microwave	$3.98*10^{-25}$	Heating up food
485 nm	Visible	$4.10*10^{-19}$	Fluorescent LM
230 nm	UV	$8.65*10^{-19}$	Fluorescent LM
20 nm	x-ray	$9.95*10^{-18}$	Crystallography
5 pm	Gamma ray	$3.98*10^{-14}$	Diagnostic PET
2-4 pm	Electron		EM
~1 Å	Neutron		Neutron diffraction

IV. Which of the above techniques/applications might be able to break single covalent bonds between carbons during illumination? (C-C bonds have an energy around ~350 kJ/mol) How could this impact an imaging experiment?

$$\frac{390\ kJ}{mol}\cdot\frac{1\ mol}{6.02\cdot10^{23}\ molecules}\cdot\frac{1000\ J}{kJ}=\frac{6.48\times10^{-19}\ J}{molecules}=6.48\times10^{-19}\ J$$

$$\lambda=\frac{hc}{E}=\frac{\left(6.63\cdot10^{-34}\ J\cdot s\right)\cdot\left(3.00\cdot10^{8}\ \frac{m}{s}\right)}{6.48\cdot10^{-19}\ J}=3.07\cdot10^{-7}\ m=307\ nm$$

Any wavelength smaller than 307 nm will cause damage to C-C bonds in the sample. That means UV, X-ray and Gamma ray radiation will cause damage to this sample.

2.(28 points) Electron accelerations

I. (6 points) What is the classical energy of an electron that initially travels at 0 m/s and has experienced a uniform 200kV? What is the speed? What is the associated de Broglie wavelength?

II. (10 points) When the speed of particle is traveling close to the speed of light, the classical description of motion isn't accurate anymore. Use the following relativistic relation, calculate the "actual" relativistic wavelength of electrons that have traveled down a 200kV potential.

$$p^2c^2 = E^2 - m_0^2c^4$$

E | total energy of particle ($E = m_0c^2 + KE$)
KE | kinetic energy of particle
c | speed of light
p | momentum
m_0 | rest mass of an object
h | Planck's constant = 6.626*10^-34 J*s
λ | wavelength
charge of electron = 1.60*10^19 C
mass of electron = 9.11*10^-31 kg

III. (12 points) The probability of scattering is proportional to the amount of time that the electrons spend in the sample. In other words, the faster the electrons travel, the less frequently they scatter. Therefore, for thicker sample imaging, people have attempted to build high voltage EM to increase the velocity of the electrons. Calculate and compare the velocities of electrons traveling down 200kV and 5MV potentials. Why is it hard to increase the speed of electrons?

PS1.Q2

1) Potential difference between filament and ground:

$$V_B - V_A = 200\,kV$$

Thus Classical energy of an electron is $E = e \times U = 3.2 \times 10^{-14}$ J

$E_{total} = P + KE \qquad KE = \frac{1}{2} m v^2$

thus $v = \sqrt{2KE/m_o} = 2.65 \times 10^8\,m/s$

$$\lambda_{DeBroglie} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\,J \cdot S}{9.11 \times 10^{-31}\,kg \times 2.65 \times 10^8\,m/s} = 2.74 \times 10^{-12}\,m$$

2) $$c^2 p^2 = (m_o c^2 + KE)^2 - m_o^2 c^4 = KE^2 + 2 m_o c^2 \times KE$$

$$P = \frac{1}{c}\sqrt{KE^2 + 2KE\,m_o c^2} = 2.63 \times 10^{-22}$$

$$P = \frac{h}{\lambda_{rel}} \qquad \lambda_{rel} = \frac{h}{P} = 2.51 \times 10^{-12}\,m$$

3) $$P = m_o v \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$ rearrange for v.

$$v = \frac{P}{\sqrt{m_o^2 + P^2/c^2}}$$

plug in P calculated from 2). $v_{200kV} = 2.08 \times 10^8\,m/s = 0.695c$

$v_{5MeV} = 2.98 \times 10^8\,m/s = 0.996c$

Due to special relativity, the faster the particle travels, the heavier it becomes. Therefore additional input in energy will have less effect in increasing the speed that e^- travelling.

3. (27 Points) Lenses and Electron Microscopes

I. (7 points) Explain how optical lenses work using the concept of optical path lengths.

Radiation scattered from one point is focused by a (convergent) lens to one point on the other side such that all possible paths through the lens have the same optical path length.

(Convergent) lenses focuses divergent radiation. Say there is one point on the left side of a lens from which photons radiate. On the opposite side of a convergent lens, there is one point where the probability of detecting a photon is highest. At this point, all radiation that has traveled through the lens converges with same optical path length, and is therefore in phase and constructively interfering. (Radiation that travels farther in air to the top of the lens travels a shorter distance through the lens; and radiation that travels shorter distances in air to the center of the lens, will travel a longer distance through the lens).

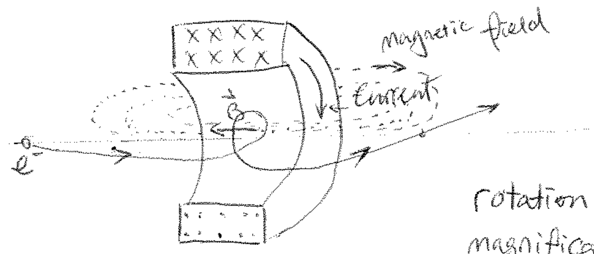
II. (10 points) What are electron lenses made of? How do electron lenses work in terms of magnification? Why do they cause image rotation as well as magnification? Use a diagram.

Electron lenses are made of a coil of wire to through which a current in applied, like a solenoid. The current in the coil creates a perpendicular magnetic field, which is used to change the path of an electron using the Lorentz force. The change in direction is the cross product of the velocity of the electron with magnetic field.

Electrons passing through the center of the lens continue though with their path unchanged. Electrons with a component of their velocity perpendicular to the magnetic field experience a force that changes their path and causes a spiral trajectory. The change in direction can be used to magnify the image, and the point in spiral where the image is measured determines the rotation.

Electron lenses are simple coils. Currents through the coil produce a toroidal magnetic field, which exerts forces on imaging electrons according to F = qv x B (all vectors). Using the right hand rule, one can see that electron diverging away from the optical axis are first displaced laterally, and then towards the axis. The lateral acceleration is “undone” by the back side of the lens, but the axial acceleration is doubled.

PS1.1 Q3. II)

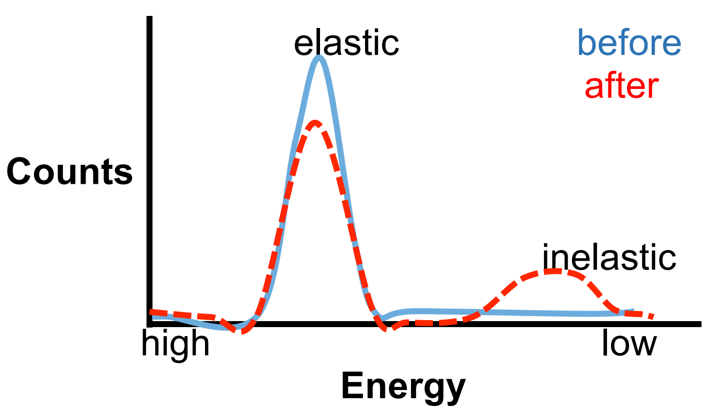


rotation : spiral trajectory
magnification : change in direction

III. (10 points) Explain briefly the optical path lengths in electron lenses.
In an EM lens, electrons that travel straight from the source through the center of the lens have no change in direction. Electrons that diverge from the source and do not pass through the center of the lens have a longer path in air to the lens and to the image point. Further, these electrons experience circumferential forces that cause the electron to travel around the optical path. Therefore, these electrons will have a longer optical path from the point of origin and the image point. As a result the electrons can interact constructively or destructively (or something in between). This will be important to consider when we talk about the contrast transfer function later.

4. (20 points) Energy filters and applications

I. (6 points) Draw on the graph the distribution of energies of electrons right before and after passing through the sample. Mark the elastically and inelastically scattered electrons.



II. (7 points) What is the function of an energy filter?

Electrons are scattered either elastically by the nucleus of the atom or inelastically by shell electrons. Elastic scatter results in no loss of electron energy, while inelastic scatter results in an element-specific loss of electron energy. The difference in energy between the elastically and inelastically scattered electrons results in a chromatic aberration of the objective lens, which in - turn compromises the quality of the resultant image. This effect is noticeable particularly as the thickness of the specimen increases.

Energy filters allows higher resolution image by forming images with energy-selected (monochromatic) electrons.

III. (7 points) Electron energy-loss spectroscopy (EELS) is an analytical technique that is based on inelastic scattering of electrons in a thin specimen. How could energy filters be used in EELS? Draw an example energy filter range on the graph. How can this kind of technique be useful in structural biology studies?

Because of element-specific loss of electron energy, with the energy filters set to collect particular energy range of inelastically scattered electrons, it can provide structural and chemical information about a specimen, even down to atomic resolution.

Reference: http://iopscience.iop.org/article/10.1088/1757-899X/109/1/012007/pdf